

DAY — **08**

SEAT NUMBER

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2025 VII 03

1100

J-384**(E)**

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs.**(8 Pages)****Max. Marks : 80****General instructions :**

The question paper is divided into **FOUR** sections.

- (1) **Section A :** Q. 1 contains **Eight** multiple choice type questions carrying **Two** marks each.
Q. 2 contains **Four** very short answer type questions carrying **One** mark each.
- (2) **Section B :** Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks.
(Attempt any **Eight**)
- (3) **Section C :** Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks.
(Attempt any **Eight**)
- (4) **Section D :** Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks.
(Attempt any **Five**)
- (5) Use of log table is allowed. Use of calculator is not allowed.
- (6) Figures to the right indicate full marks.
- (7) Use of graph paper is not necessary. Only rough sketch of graph is expected.

- (8) For each multiple choice type of questions, only the first attempt will be considered for evaluation.
- (9) Start answer to each section on a new page.

SECTION – A

Q. 1. Select and write the correct answer of the following multiple choice type of questions : [16]

- (i) The inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is _____
- (a) $(p \wedge q) \rightarrow (p \vee q)$
 (b) $\sim(p \vee q) \rightarrow (p \wedge q)$
 (c) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$
 (d) $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$ (2)
- (ii) In $\triangle ABC$, if $a=2$, $b=3$ and $\sin A = \frac{2}{3}$, then $\angle B =$ _____
- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$ (2)
- (iii) If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 7\hat{k}$ and initial point $A \equiv (1, 5, 0)$ then terminal point B is _____
- (a) $(1, 3, 7)$ (b) $(7, 3, 1)$
 (c) $(1, 7, 3)$ (d) $(3, 1, 7)$ (2)
- (iv) The angle between the lines
 $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and
 $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ is _____
- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
 (c) $\frac{\pi}{3}$ (d) 0 (2)

- (v) If y is a function of x and $\log(x + y) = xy$ then the value of $\left(\frac{dy}{dx}\right)$ at $x = 0$ is ____
- (a) 1
(b) -1
(c) 2
(d) 0
- (2)

- (vi) If the displacement of a particle at time t is given by $S = 2t^3 - 5t^2 + 4t - 3$, then its acceleration at time $t = 1$ is ____
- (a) 2
(b) 8
(c) 10
(d) 14
- (2)

- (vii) The solution of the D.E. $\sec^2 x \cdot \tan y dx + \sec^2 y \cdot \tan x dy = 0$ is ____
- (a) $\tan x \cdot \cot y = c$
(b) $\cot x - \cot y = c$
(c) $\tan x \cdot \tan y = c$
(d) $\cot x - \tan y = c$
- (2)

- (viii) If X is waiting time in minutes for a bus and its p.d.f. is given by $f(x) = \frac{1}{5}$, for $0 \leq x \leq 5$,
 $= 0$, otherwise
then the probability that waiting time is between 1 and 3 is ____
- (a) $\frac{1}{5}$
(b) $\frac{2}{5}$
(c) $\frac{3}{5}$
(d) $\frac{4}{5}$
- (2)

Q. 2. Answer the following questions : [4]

(i) Find the general solution of $\tan \theta = 0$. (1)

(ii) Find the magnitude of the vector $\vec{a} = 3\hat{i} + \hat{j} + 7\hat{k}$ (1)

(iii) Find $\frac{dy}{dx}$, if $y = \sin(\log x)$. (1)

(iv) Evaluate : $\int \frac{1}{\sqrt{x}} dx$. (1)

SECTION – B

Attempt any EIGHT of the following questions : [16]

Q. 3. Using truth table, prove that $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$ (2)

Q. 4. Find the cofactors of the elements of the matrix $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$ (2)

Q. 5. Find the polar coordinates of the point whose cartesian coordinates are $(-\sqrt{2}, -\sqrt{2})$. (2)

Q. 6. In $\triangle ABC$, if $a = 2$, $b = 3$, $c = 4$, then prove that the triangle is obtuse angled. (2)

Q. 7. If $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, then find $|\vec{a} \times \vec{b}|$ (2)

Q. 8. Find the cartesian equation of the plane passing through the point $A(-1, 2, 3)$, the direction ratios of whose normal are 0, 2, 5. (2)

Q. 9. Find $\frac{dy}{dx}$, if $x\sqrt{x} + y\sqrt{y} = a\sqrt{a}$. (2)

Q. 10. Show that the tangent to the curve $y = x^3 - 6x^2 + x + 3$ at the point $(0, 3)$ is parallel to the line $y = x + 5$. (2)

Q. 11. Check whether the conditions of Rolle's theorem are satisfied by the function $f(x) = x^2 - 4x + 3, x \in [1, 3]$ (2)

Q. 12. Evaluate : $\int \frac{dx}{x+x^{-10}}$ (2)

Q. 13. Evaluate : $\int_0^{-1} e^{-x} dx$ (2)

Q. 14. If $X \sim B(n, p)$ and $E(X) = 6, Var(X) = 4.2$, then find n and p . (2)

SECTION – C

Attempt any EIGHT of the following questions : [24]

Q. 15. In $\triangle ABC$, prove that $a^2 = b^2 + c^2 - 2bc \cos A$ (3)

Q. 16. Find the combined equation of pair of lines passing through $(2, 3)$ and perpendicular to the lines $3x + 2y - 1 = 0$ and $x - 3y + 2 = 0$. (3)

Q. 17. Show that the acute angle θ between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is given by,

$$\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right| \quad (3)$$

Q. 18. Using vector method, prove that the medians of a triangle are concurrent. (3)

Q. 19. Find the vector equation of the plane passing through the point $A(-1, 2, -5)$ and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$. (3)

Q. 20. Find the shortest distance between the lines, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$
 and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. (3)

Q. 21. Water is being poured at the rate of $27 \text{ m}^3/\text{sec}$ into a cylindrical vessel of base radius 3 m. Find the rate at which the water level is rising. (3)

Q. 22. Evaluate : $\int \frac{\sin(x+a)}{\cos(x-b)} dx$ (3)

Q. 23. Solve the D.E.
 $\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$ (3)

Q. 24. Obtain the differential equation by eliminating the arbitrary constants from $y = c_1 \cos(\log x) + c_2 \sin(\log x)$. (3)

Q. 25. The probability distribution of X is as follows :

| | | | | | |
|----------|-----|-----|------|------|-----|
| x | 0 | 1 | 2 | 3 | 4 |
| $P[X=x]$ | 0.1 | k | $2k$ | $2k$ | k |

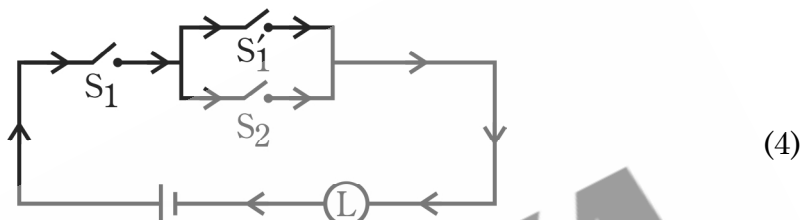
Find (i) k , (ii) $P(X < 2)$, (iii) $P[1 \leq X < 4]$ (3)

Q. 26. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four correct answers just by guessing? (3)

SECTION – D

Attempt any FIVE of the following questions : [20]

- Q. 27.** Give an alternative equivalent simple circuit for the following switching circuit :



- Q. 28.** The sum of three numbers is 2. If twice of the second number is added to the sum of first and third number we get 0. Adding five times the first number to twice the sum of second and third number we get 7. Find the numbers using matrix method. (4)

- Q. 29.** Using properties of scalar triple product, prove that
 $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$ (4)

- Q. 30.** Solve the following L.P.P. using graphical method :
 Maximize, $z = 9x + 13y$
 Subject to, $2x + 3y \leq 18,$
 $2x + y \leq 10$
 $x \geq 0, y \geq 0$ (4)

- Q. 31.** If $x = f(t)$ and $y = g(t)$ are differentiable functions of t , so that y is a function of x and $\frac{dx}{dt} \neq 0$, then prove that

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

- Hence find $\frac{dy}{dx}$, if $y = at^2$ and $x = 2at$. (4)

Q. 32. Prove that : $\int \sqrt{a^2 - x^2} \cdot dx = \frac{x}{2} \cdot \sqrt{a^2 - x^2} + \frac{a^2}{2} \cdot \sin^{-1} \left(\frac{x}{a} \right) + c$ (4)

Q. 33. Evaluate : $\int_0^{\frac{1}{2}} \frac{dx}{(1-2x^2) \cdot \sqrt{1-x^2}}$ (4)

Q. 34. Find the area of the region lying between the parabolas $y^2 = 4x$ and $x^2 = 4y$. (4)